The architecture of the Forty-Eight

Peter Bradley-Fulgoni

'Pythagoras showed that there was a flaw in nature's tuning'

The Forty-Eight, the colloquial name for J. S. Bach's *Das Wohltemperierte Klavier* (the well-tempered clavier) consists of 48 pairs of pieces, each pair made up of a prelude followed by a fugue. They are divided into two books. The 24 in each book follow the 12 notes, the semi-tones, of the chromatic scale (C, C sharp, D, D sharp, E, F, F sharp, G, G sharp, A, A sharp, B; then you get back to C again) and for each note you have a major and a minor key. So, for instance, in Book 1 of the Forty-Eight, the first pair is in C major, the next in C minor and so on. Then you have a second book of the same.

These works, written in the 1720s, were a manifesto for equal temperament, or adjustments in keyboard tuning. Before Bach's time, temperament was based on the Pythagorean proportions of harmonies, in turn based on observations in nature. In *Experiencing Architecture,* Rasmussen tells the story of Pythagoras hearing pleasing harmonies as he passed a blacksmith and returning to investigate. He discovered that the lengths of the three hammerheads were related to each other in the ratio of 6:4:3. The longest produced the keynote; the pitch of the shorter was a fifth above it and that of the shortest an octave above it. In the same way, the shortening of a vibrating string increases its pitch. The key harmonic relationships here are the octave, obtained by halving the length of a string thus doubling its frequency of vibration, and the fifth, obtained by shortening the string by a third and increasing its frequency by 3/2 or 1.5. But by applying mathematical values to sound Pythagoras also showed that there is a flaw in nature's tuning: its fifths are a little out with its octaves.

The octave is the perfect interval: the ratio of the note and its octave is 2; double the frequency and you get the same note an octave higher. Theoretically you could go on forever like that. On a modern keyboard, if you move from A to A through the seven octaves until you reach the highest A, and with every jump you double the frequency, you have a progression from x to $2x$ through 4, 8, 16, 32, 64, and finally to $128x$. After octaves, the easiest interval to tune is the fifth (you are forever hearing it with stringed instruments). On the piano you get 12 fifths, from A to E, then B, F sharp, C sharp (or D flat), A flat, E flat, B flat, F, C, G, D and finally to top A. In each case the frequency of vibration will become one and a half times as many. Unfortunately 1.5 multiplied up (1.5 to the power of 12) does *not* arrive at 128 (2 to the power of 7), but slightly more and, on this basis, would result in the tuning of the top A being slightly sharp. This pitch error (about a quarter of a tone on a piano) is known as the Pythagorean comma.

While the compass of keyboard instruments was quite small, things didn't sound too disagreeable. But as composers extended the range of their own creativity, their creative palette increased accordingly and required instruments to follow suit. Then, in the higher octaves, one could hear the Pythagorean comma (and considerable historical and archaeological evidence corroborates this dilemma of the time because the tuning of organs of the period was sometimes left unchanged until our own times). The necessity arose for an

adjustment that ironed out this natural discrepancy and that is what equal temperament is.

Pythagoras, with only a monochord or a dulcimer, made the mathematical point about the octave and the fifth. But if one thinks of all the other intervals, tuning by A not only gives two different readings for top A, it makes a distinction between many other notes as well, for example A flat and G sharp. Of the various practical solutions proposed, the one that triumphed was the idea of tuning 12 exactly equal semi-tones in an octave. Every single semi-tone progression was smoothed over by taking a given note and (as it were) multiplying its frequency by the 12th root of two (1.05946), then taking the resultant frequency and doing the same again and so on.

The results are that there is no distinction between A flat and G sharp, which are enharmonically the same on a modern piano but continue to be minutely distinct on a stringed or a wind instrument, and that all scales are equally out of tune – to most ears imperceptibly out of tune. Every fifth as one rises is slightly flat. This resolution has ironed out a lot of tone colour – a sort of acoustic atmosphere that was peculiar to specific keys – less readily recognisable now as the sound of a particular key is far blander than it was. In an extreme case – where someone tunes a piano electronically, multiplying every frequency by the 12th root of two to get the next semi-tone up – the instrument sounds pretty boring. A clever tuner counteracts this tendency by rule of thumb, perhaps tuning a little sharp in the lower bases. Horowitz, for example, used to like everything on the sharp side. The object of this tweaking is to enhance the tone colour by departing from the purely mathematical and rather arid experiment that is the essence of equal temperament.

We have grown up with the sound world of equal temperament and it sounds right. At the time it was introduced, however, it was quite an aberration and much disputed. Bach's set of Forty-Eight preludes and fugues in every possible key settled the argument. Of course it was more than a manifesto or a celebration of a great idea. He did not write for a more extensive keyboard just to prove his point. Necessity here was the mother of invention; he wanted to make a musical statement, needed a more extensive keyboard compass for it, and therefore needed also a tuning that was consistent for that keyboard. One suspects that, like many artists, he also enjoyed working within – and exceeding – the boundaries the system set. And perhaps this discipline is also why so many great composers adopted Bach's format: Chopin is just one example; his 24 preludes go through all the keys and the opus 10 book of his 24 studies follows each major key by the relative minor.

The general adoption of equal temperament paved the way for the sonata first movement principle, an essentially dramatic form in which the first and second subject groups are in contrasting keys and the development section is free to modulate in a more far-reaching manner. Thematic material could be more widely spaced on the keyboard but there was now no fear of dissonance in higher registers. Bach's son, C. P. E. Bach, is credited with the sonata system which was then brought to near perfection by Haydn, Mozart and, later, with ever-greater freedom and iconoclasm, by Beethoven. And with the onset of that freeing of the boundaries of the sonata principle came Romanticism. In spite of its mistuned intervals, equal temperament made possible the continued use of a keyboard with 12 notes to the octave and the

Johann Sebastian Bach: writing for a more extensive keyboard ...

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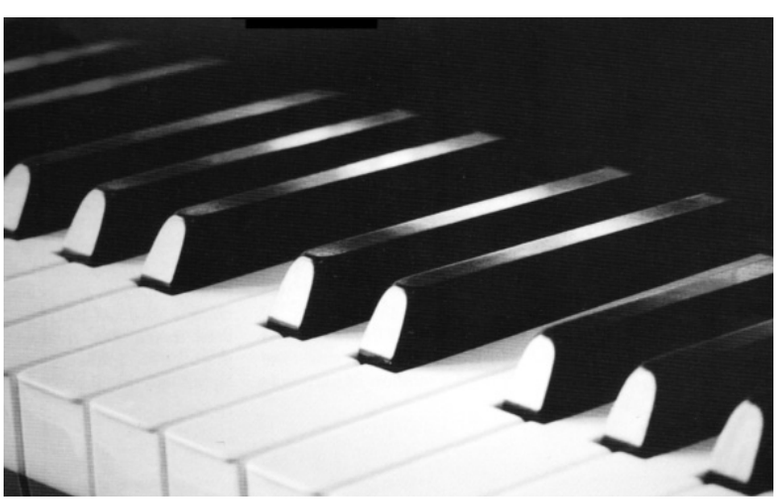
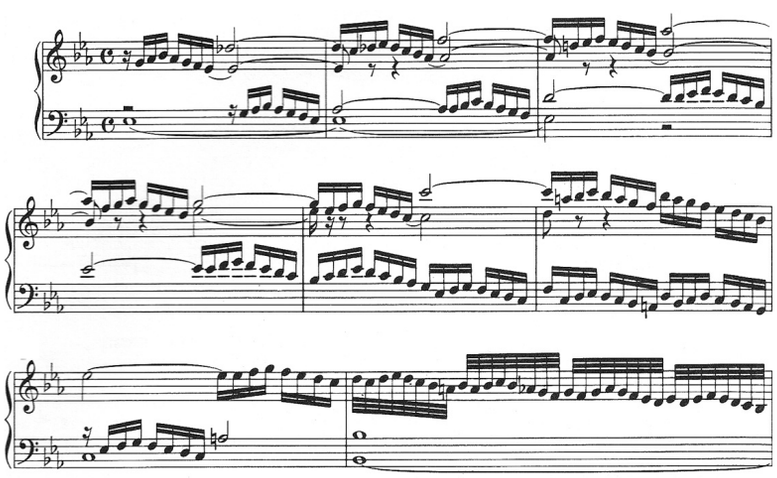


... which made audible the Pythagorean comma

development of nineteenth-century piano music and technique in the hands of figures such as Franz Liszt. Bach was the trail-blazer.

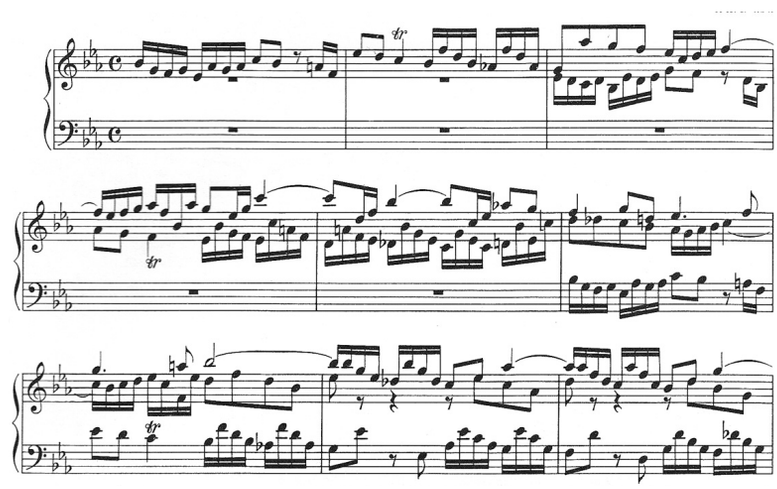
As for the preludes and fugues themselves, the prelude could have been anything. It is determined by its function, namely to precede a fugue. The fugue is determined by its style rather than by its function: it is a texture with a highly disciplined format. The juxtaposition of the prelude's rather free-flowing, sometimes improvisatory music being followed by something quite rigorous, strict and disciplined gives an emotional polarity. But here they are wedded to sentiment, feeling and emotion and that is what makes Bach's preludes and fugues not only very interesting structurally and formally, but extra special because there is what one might call an almost Romantic spirit of expressiveness, an emotional content which seems to come from disciplined formal conditions – an unusual and paradoxical dichotomy. Some of the fugues have more than one subject. Take the F sharp minor fugue in book 2: three subjects blend at some point in the fugue, like a matrix of genius; three ideas which, when sampled individually, seem disparate and diverse, when they come together don't just harmonise, they coalesce perfectly in the structure of the fugue, making the whole process a crescendo of creative power. It is a remarkable moment. Some of the preludes are in simple binary form, but others are fugues in themselves; an example is the E flat prelude in book 1, a very sophisticated fugue, more complex than the 'official' one that follows it. What provides for all this precisely is the genius of Bach, using the formal structures of his epoch to the *n*th possible degree and drawing every ounce from a principle. He was characterised by his free-flowing imaginative process, not a confinement to rule. His art raises the question 'what is Romanticism?' and always suggests that the answer can no longer be confined to a period post-Beethoven.

It is also best played on a keyboard that is slightly out of tune.



J. S. Bach, Prelude in E flat, from book 1 of the Forty-Eight: a more complex fugue than the 'official' one that follows it

'an emotional content that seems to arise from disciplined formal conditions'



J. S. Bach, Fugue in E flat from Book 1 of the Forty-Eight

Peter Bradley-Fulgoni studied piano in England, Italy and Russia. He has made many broadcasts, given concerto and recital performances throughout Europe and beyond and recorded a number of CDs. Recognised as an inspirational teacher, a DVD featuring both musical performance and discussion will be released by Sound Techniques in their Piano Maestros series in Spring 2007 (www.soundtechniques.co.uk)